Theorem: $\binom{n}{k} = \frac{n!}{(n-k)!\,k!}$

$$\underbrace{\binom{n}{k}}_{\substack{\text{\# of k-element} \\ \text{subsets of an n-element} \\ \text{set}}} = \underbrace{\binom{n-1}{k}}_{\substack{\text{\# of k-element} \\ \text{subsets of an} \\ \text{(n-1)-element set}}} + \underbrace{\binom{n-1}{k-1}}_{\substack{\text{\# of (k-1)-element} \\ \text{subsets of} \\ \text{an (n-1)-element} \\ \text{set}}}$$

Let $S = \{1, \ldots, n\}$

The LHS, $\binom{n}{k}$ is the number of k-element subsets of S.

Notice that $\binom{n-1}{k}$ is the number of k-element subsets for $S' = \{1, \ldots n-1\}$.
Any k-element subset of X of S either includes n or does not. In $n \notin X$, then X is a subset of size k of an $(n-1)$ element set. In $n \in X$, then $X \setminus \{n\}$ is a $(k-1)$-element subset of an $(n-1)$-element set.

This yields the identity $\square$